FALL 2019: MATH 558 QUIZ 6

Each question is worth 2 points.

(i) Give an example of a ring which is not a commutative ring.

Solution. The ring of 2×2 matrices over \mathbb{R} is not commutative.

(ii) Give an example of a commutative ring different from the integers.

Solution. Any field, e.g., \mathbb{R} is a commutative ring, as is any polynomial ring with coefficients in a field.

(iii) Give an example of a ring in which there are non-zero elements a, b such that $a \cdot b = 0$.

Solution. \mathbb{Z}_4 , with $\overline{0} = \overline{2} \cdot \overline{2}$. Similarly, \mathbb{Z}_n , with *n* not prime, also works.

(iv) Give an example of an integral domain that is not a field.

Solution. \mathbb{Z} or any polynomial ring with coefficients in a field.

(v) Give an example of a field.

Solution. $\mathbb{Q}, \mathbb{R}, \text{ or } \mathbb{C}$.